Josephson Arbitrary Waveform Synthesizer (JAWS) as a quantum standard of voltage and current harmonics

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The operation of Josephson voltage standards can be extended to applications other than ac voltage measurements by using:

- voltage dividers to extend their voltage range
- current transformers or current shunts to extend the mode of operation
- voltage harmonics can be traced to the SI through a primary realisation

Take home message
Take home message (... continued)

- NIST Standard Reference Instruments
  
  JAWS chip + driving electronics + software

Take home message (… continued)

- Extent the voltage range to 1000 V
- Extent the operation to current (200 A)
- Improve connectivity (unload the JAWS)

1 V - 2 V
Traceability and Calibration

SI units

<table>
<thead>
<tr>
<th>Traceability</th>
<th>Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>National standards (NMI)</td>
<td></td>
</tr>
<tr>
<td>Reference standards</td>
<td></td>
</tr>
<tr>
<td>Working standards</td>
<td></td>
</tr>
<tr>
<td>Measuring equipment</td>
<td></td>
</tr>
<tr>
<td>Everyday life measurements</td>
<td></td>
</tr>
</tbody>
</table>
Traceability example for dc voltage

SI volt

5000 µV/V

12 µV/V

5 µV/V

1 µV/V

<0.001 µV/V
What we do at NMI?

$$(A \pm B)_U$$

Value \hspace{2cm} Uncertainty \hspace{2cm} Unit

<5 nV/V @ 10 V
Outline

- Motivation
- System operation
- Evaluation
- Uncertainty
- Conclusion
Conventional calibration of power analyzers

Signal

Power analyzer under test (IUT)

Reference power analyzer

Or

Reference source

Power analyzer under test (IUT)
Motivation

Main Objective
Provide traceability to spectrum analysers and harmonic sources to the SI for both the magnitude and the phase
Different frequency ranges classified by IEC for distribution networks
Conventional methods for the traceability of harmonics

\[ A_1 \sin(\omega_1 t) + ... A_k \sin(\omega_k t) ... + A_n \sin(\omega_n t) \]

\[ A_k \sin(\omega_k t) \]

\[ ab = \frac{(a + b)^2 - (a - b)^2}{4} \]
NMIA Thermal Power Comparator

Controlled Switches
Amplifiers

Temperature Compensation

Outputs

Differential Sum-and-Difference

Controlled Switches
Differential Amplifiers

Temperature Compensation
What is wrong with these standards? Why JAWS?

- Fundamentally, nothing wrong (at least for magnitude)
- Practically,
  - Low uncertainty for magnitude
  - High uncertainty for phase

- Why JAWS
  1. Fundamental realization of the volt in the new SI
  2. Low uncertainty for magnitude and phase
  3. Verify the operation of the conventional standards using another operating principle
Target uncertainties

- Target uncertainty for magnitude: 0.0005 % of fundamental
- Target uncertainties for phase
  - 0.001° for harmonics 3 to 7
  - 0.005° for harmonic up to 39
Outline

• Motivation
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Josephson voltage standards:
inverse ac Josephson effect

\[ V = n \frac{h}{2e} f \]

Insulator (SIS)

Normal metal (SNS)

5 µV to 150 µV per junction
JVS agree better than 1 part in $10^{10}$
Josephson Arbitrary Waveform Synthesiser (JAWS)

JJA: Josephson Junction Array
Modulate a waveform using pattern of rf pulses to bias the JJA
Quantised voltage pulses

For each junction

\[
\int_{t_i}^{t_{i+1}/f_s} v(t) \, dt = \frac{h}{2e}
\]
Josephson Arbitrary Waveform Synthesizer at NMIA

- 4 × 12800 junctions
- Max dc voltage 0.4 V
- Operating CW frequency: 15 GHz

- Lowest Signal Frequency: 56 Hz
- 250 mV_{rms} per JJA
JAWS set up
Quantum NMI logo
Quantum NMI logo

The Quantum NMI Logo - Spectral distribution of 43 precisely known harmonics of 400 Hz generated with Josephson Arbitrary Waveform Synthesizer
Real life waveforms can be distorted

Current waveform measured in an LED type lamp
How do we use the JAWS?

Extent the voltage range to 1000 V

Extent the operation to current (200 A)

Improve connectivity (unload the JAWS)

< 250 mV
Calibration of power analyzers (meters)

Voltage

10 MHz

Josephson Arbitrary Waveform Synthesizer

Instrument Under Test

<250 mV

10 MHz

AWG

Voltage amplifier

Instrument Under Test

Inductive Voltage Divider

Digital Sampling Voltmeter

Josephson Arbitrary Waveform Synthesizer

>250 mV

$H(jf)$

$P(jf)$
NMIA Inductive Voltage Divider

Max input voltage:
1100 V rms or 20 V/Hz
Nominal frequency range:
40 Hz to 1 kHz
Typical ratio errors at 50 Hz and 60 Hz
in-phase: 1 part in $10^9$ of input
quadrature: 5 parts in $10^9$ of input
Calibration of meters (... continued)

Current

10 MHz → AWG → Transconductance amplifier → Multi-range Current Transformer → Digital Sampling Voltmeter → IUT

Josephson Arbitrary Waveform Synthesizer
Current shunts

Up to 20 A, 100 kHz

Typical phase error: less than 3 $\mu$rad at 1 kHz, less than 10 $\mu$rad at 10 kHz
Multi-range current transformer

Current ranges: 0.125 A to 200 A
Ratio error (mains frequency):
    less than 2 parts in $10^6$
Phase displacement (mains frequency):
    less than 3 µrad
Frequency: 50 Hz to 1 kHz, extended to 10 kHz
Link of JAWS to volt and rad

volt (magnitude: $a+ib$)  rad (phase angle: $a+ib$)

$h/2e$ (flux quantum) + time (s)

Josephson Arbitrary Waveform Synthesizer

+ IVD + current shunt (or current transformer)
Outline

• Motivation
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Josephson Arbitrary Waveform Synthesiser (JAWS)

For each junction

\[ \int_{t_i}^{t_i+1/f_s} v(t) dt = \frac{h}{2e} \]
JAWS evaluation as harmonic standard

VNIIM sampling system
Aperture time: 50 μs
Sample period: 70 μs

NI PXI 5922
Sampling frequency: 100 kHz
How to demonstrate that the JAWS is a fundamental standard

The generated waveform is not affected by
- specific JJA
- biasing electronics
- repetition frequency and the rf pattern
- compatible with common sense
Sine waveform test: Comparison of JAWS with the ac-dc transfer measurements (agreement better than 2 µV/V, up to 5 kHz)
Typical test signals

- Signal frequency: 60 Hz
- Harmonics: 3, 5, 7, 9, 11, 23, 31, 39
- 150 - 200 mV\textsubscript{rms}
Typical test signals (… continued)
Test waveforms

<table>
<thead>
<tr>
<th>Harmonic number</th>
<th>Magnitude (%)</th>
<th>Phase (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>23</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>31</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>39</td>
<td>10</td>
<td>90</td>
</tr>
</tbody>
</table>
Common sense: Magnitude of spectrum (... continued)

- Distorted waveform test:
  Comparison of the JAWS with the thermal power standard
  (agreement better than 0.001% of the fundamental)
Phase difference between two JJA single tone measurements, 158 mV rms

Two JJAs
Different rf path (amplifiers + attenuators + dc blocks)
Different low frequency compensation

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Phase error (°)</th>
<th>Standard deviation (°)</th>
<th>Transfer standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.00003</td>
<td>0.00051</td>
<td>VNIIM</td>
</tr>
<tr>
<td>400</td>
<td>0.00030</td>
<td>0.00006</td>
<td>VNIIM</td>
</tr>
<tr>
<td>1000</td>
<td>0.00019</td>
<td>0.00005</td>
<td>VNIIM</td>
</tr>
</tbody>
</table>
How to demonstrate that the JAWS is a fundamental standard

The generated waveform is not affected by

- specific JJA
- biasing electronics
- repetition frequency and the rf pattern
Effect of JJAs and biasing electronics
Different JJA, same electronics

Diagram:
- CW Generator
- Ternary Pattern Generator
- AWG
- V/I converter
- Sync
- Data
- JJA1
- JJA2
- 4.2 K
- V1
- V2
- 10 MHz
Effect of JJAs and biasing electronics
Different JJA, same electronics

Phase error (°)

Harmonic number

• JJA1
• JJA2
How to demonstrate that the JAWS is a fundamental standard

The generated waveform is not affected by

- specific JJA
- biasing electronics
- repetition frequency and the rf pattern
Effect of JJAs and biasing electronics
Different JJA, different electronics

![Graph showing phase error vs harmonic number for JJA1 and JJA2]
How to demonstrate that the JAWS is a fundamental standard

The generated waveform is not affected by

- specific JJA
- biasing electronics
- repetition frequency and the rf pattern

\[ f_{s1} \]
\[ f_{s2} \]
Different repetition frequencies and rf patterns

-0.0030  -0.0020  -0.0010  0.0000  0.0010  0.0020  0.0030

Phase error (°)

Harmonic number

13.9776 GHz
15.0528 GHz

Measurement.gov.au
Outline

• Motivation
• System operation
• Evaluation
• Uncertainty
• Conclusion
Sources of uncertainty in JAWS

- LF compensation
- rf bias
  - 10 MHz
- JJA
- Measuring electronics
Transmission line

\[ L_s \]

\[ R \quad L \]

\[ G \quad C \]

\[ R_L \quad C_L \]
Transmission line (take 2)

Both are approximations (have errors)

Difference between models: 5-6 parts in $10^8$
Total target uncertainty: 10 parts in $10^8$
Discrepancy between models
Our approach
Transmission line equations
(distributed parameters)
Our approach on the transmission line

Dominant at $f > 10$ kHz
Sources of uncertainty in JAWS

- LF compensation
- rf bias
- Measuring electronics

10 MHz
Parasitic on-chip inductance

Errors due to:
• LF compensation current
• rf pulses
Chip inductance: effect on compensation signal

\[ i_c(t) = I_c \sin(\omega t) \]

\[ v_{Li}(t) = L_i \frac{di_c(t)}{dt} \]

\[ v_i(t) = \omega L_i \cos(\omega t) \]

\[ \sum v_{Li} \]

\[ \sum v_{JJAi} \]

Dominant at \( f < 10 \text{ kHz} \)
Chip delays and rf lines: effect on rf pulses

Pulse delay + dispersion (pulses stretched)

$t_{pd} \geq t_p$
Other errors of JAWS

Frequency
• bias
• phase noise (variability in pulse density per unit time)

Δ-Σ modulation

rf power coupled at the output

… even more errors? (second order)
What other uncertainty components?

- Voltage and current transducers:
  - magnitude uncertainties
  - phase uncertainties
- Loading errors when putting the parts together
Loading effects when putting the system together

\[ X(i\omega) \quad H(i\omega) \quad Y(i\omega) \]

no loading \[ Y(i\omega) = X(i\omega) H(i\omega) \]

loading \[ H_1(i\omega) \neq H(i\omega) \]
\[ Y(i\omega) \neq X(i\omega) H(i\omega) \]
Example: loading of the IVD output by the DSVM

\[ V_{DVM} = V_{IVD} \frac{Z_{DVM}}{Z_{DVM} + Z_0} \]

if \( |V_{DVM} - V_{IVD}| \leq \varepsilon \)

As \( \varepsilon \to 0 \), \( Z_{DVM} \geq \frac{1}{\varepsilon} Z_0 \)

\( \varepsilon = 1 \mu V \) \( Z_{DVM} > 10^6 Z_0 \)
## Uncertainty budget

### UNCERTAINTY CALCULATION - 0.154 V, 10 %, up to 10th

<table>
<thead>
<tr>
<th>Item</th>
<th>COMPONENT</th>
<th>TYPE</th>
<th>DIST</th>
<th>Expanded Uncert (SR or U 95%)</th>
<th>$k$</th>
<th>DOF</th>
<th>Std Uncert $u(x_i)$, (1s)</th>
<th>UNIT</th>
<th>Sensitivity Coefficient $\frac{\partial y}{\partial x_i}$</th>
<th>$u_i$ (nV / V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Code generation error</td>
<td>B</td>
<td>GAU</td>
<td>0.0001</td>
<td>1.0000</td>
<td>100</td>
<td>0.0001</td>
<td>°</td>
<td>1</td>
<td>0.0001</td>
</tr>
<tr>
<td>2</td>
<td>NI resolution</td>
<td>A</td>
<td>GAU</td>
<td>0.0003</td>
<td>1.0000</td>
<td>30</td>
<td>0.0003</td>
<td>°</td>
<td>1</td>
<td>0.0003</td>
</tr>
<tr>
<td>3</td>
<td>Maximum deviation between different experiments</td>
<td>B</td>
<td>GAU</td>
<td>0.0005</td>
<td>1.0000</td>
<td>10</td>
<td>0.0005</td>
<td>°</td>
<td>1</td>
<td>0.0005</td>
</tr>
<tr>
<td>4</td>
<td>rf delay</td>
<td>B</td>
<td>GAU</td>
<td>0.0001</td>
<td>1.0000</td>
<td>30</td>
<td>0.0001</td>
<td>°</td>
<td>1</td>
<td>0.0001</td>
</tr>
<tr>
<td>5</td>
<td>Cable length</td>
<td>B</td>
<td>GAU</td>
<td>0.0001</td>
<td>1.0000</td>
<td>30</td>
<td>0.0001</td>
<td>°</td>
<td>1</td>
<td>0.0001</td>
</tr>
<tr>
<td>6</td>
<td>Rounding</td>
<td>A</td>
<td>REC</td>
<td>0.0001</td>
<td>1.7321</td>
<td>100</td>
<td>0.0001</td>
<td>°</td>
<td>1</td>
<td>0.0001</td>
</tr>
<tr>
<td>7</td>
<td>Std Dev of Measurements</td>
<td>A</td>
<td>GAU</td>
<td>0.0006</td>
<td>1.0000</td>
<td>19</td>
<td>0.0001</td>
<td>°</td>
<td>1</td>
<td>0.0001</td>
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<tr>
<td>8</td>
<td>Phase resolution</td>
<td>A</td>
<td>REC</td>
<td>0.0001</td>
<td>1.7321</td>
<td>30</td>
<td>0.0000</td>
<td>°</td>
<td>1</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Combined Standard Uncert, $u_c = 0.0006$ °

Effective Total DOF, $n = 24.5153$

95% Coverage Factor, $k = 2.0660$

95% Expanded Uncert, $U = 0.0013$ °

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**JCGM 100: 2008 - Evaluation of measurement data - Guide to the expression of uncertainty in measurement (GUM)**
Calibration of power analyzers at NMIA

Signal

- Power analyzer under test (IUT)
- Thermal Power Comparator

Or

JAWS

Power analyzer under test (IUT)
Summary

• JAWS can be used for the calibration of the magnitude and the phase of harmonics
• System capability
  – 10 mV to 240 V
  – 5 mA to 20 A
  – fundamental 60 Hz, harmonics: odd up to the 39th
• Best phase uncertainties from 0.001° to 0.010° (k = 2.0)
• Accredited service ISO 17025 (NATA, ILAC equivalent to CALA)
• Future plan:
  – extend the frequency range
  – apply the technique to other applications
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